Portuguese Study Groups’ Reports

Report on
“Feasibility Study of
Housing First Project in Aveiro”

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Executive summary

The non-profit organisation Florinhas do Vouga support homeless people by providing them with basic necessities such as food, as well as helping them address long term issues, such as mental health problems. This aid is usually provided whilst a homeless person is on the street or in some form temporary accommodation. The Housing First approach is to, instead, first provide a homeless person with their own home, and then, in this more stable setting, provide further support. The social benefits of giving homeless people the opportunity to live in a house are clear. To the individual it brings stability and makes it much easier to find a job, and access other forms of provision, such as healthcare and drug rehabilitation programs. To society at large it reduces the number of people living on the street, potentially increasing tourism and economic productivity in a city’s commercial centre.

The problem is to compare the costs incurred by both the Usual (U) and Housing First (HF) models and determine whether Housing First is a financially viable alternative. The housing status of each homeless person is modelled (for example if they are on the street or in a shelter) and this is used to calculate the costs of each approach. In the simplest model, the state of each person is assumed to be constant with time. The changing state of the homeless population is then modelled (for example how many homeless people are on the streets as opposed to in temporary accommodation or a house at any one time). This is done using a Markov chain approach, and also by direct simulations which incorporate more complex constraints on the state of the homeless population. The different mathematical models provide evidence that the financial costs of Housing First are less than those of the Usual model, especially in the long term (a 36 month period). Recommendations for further work are also given.
1 Introduction

1.1 Background

Homelessness has been recognized as a growing social and public health problem in developed countries, with several cities facing a growing homeless population. Homeless individuals consume costly public health, social, and legal services. Providing care to those with severe mental illness and addiction issues is particularly challenging [4].

1.2 Usual model in Aveiro

In Aveiro, it is estimated that there are about 60 homeless people. The non-profit organisation Florinhas do Vouga make food, economic aid and psychosocial support (for mental health and substance abuse problems) available to these people. Currently this aid is provided in a compartmentalised way, as a homeless person transitions between being on the street, in an emergency shelter or in more permanent accommodation such as a hostel. We name this practice the Usual model (U). This transitional model has not resulted in people’s access to individualized and integrated housing in the community; being, therefore, ineffective in the resolution of homeless situations.

1.3 Housing First model

Housing First is a relatively recent innovation in human service programs and social policy regarding treatment of the homeless and is an alternative to the Usual model (U). Its approach is to, instead, first provide a person with their own home, integrated in mainstream neighbourhood contexts of the community, and then, in this more stable setting, provide further social support. We name this new practice as the Housing First model (HF).

Housing First approaches are based on the concept that a homeless persons’ first and primary need is to obtain stable housing, and that other issues that may affect the household can and should be addressed once housing is obtained. This model offers permanent, affordable housing as quickly as possible for individuals experiencing homelessness, and then provides the supportive services and connections to the community-based supports people need to keep their housing and avoid returning to homelessness.

Principles of Housing First are to move people into housing directly from streets and shelters without preconditions of treatment acceptance or compliance and embraces harm-reduction approach to addictions rather than mandating abstinence. In addition, homeless people in the Housing First model access support from a multidisciplinary team, following a well-defined intensive program or assertive community treatment depending on their needs.
The “Housing First” Program was launched in Los Angeles, California in 1988, in response to an increase in the number of homeless and has been implemented with great success [1, 3, 4]. Compared to the Usual model, the Housing First model reports greater residential stability and fewer arrests, greater control over drug and alcohol use, better health outcomes and wellbeing, and lower residential and health costs [1, 3, 4]. The Denver Housing First Collaborative, in USA, operated by the Colorado Coalition for the Homeless, provides housing through a Housing First approach to more than 200 chronically homeless individuals. A 2006 cost study documented a significant reduction in the use and cost of emergency services by program participants as well as increased health status.

The project “Casas Primeiro” in Lisbon (see http://www.aeips.pt/ and http://jornelas.aeips.pt/?page_id=40) made it possible for around 50 people with severe mental illness and homelessness for several years (62% for more than 6 years) to move from the streets to an individualized house, stable (not transitory) and integrated in the city. The results of the evaluation of implementation of the project demonstrate that, with individualized support tailored to specific needs, people are able to achieve a stable housing situation. Indeed 90.5% of the participants stayed in their homes, a very positive and significant value, consistent with the results evidenced in the international literature on this model. The overwhelming majority of project participants were very satisfied with their home and the support services received, perceiving significant improvements in their quality of life. Participants reported that the house provided an improvement in their personal safety, eating habits, rest, stress levels, mental and physical health. The results of the evaluation also indicate a decrease in alcohol and drug consumption. The evaluation also made it possible to verify a drastic reduction in the use of emergency services, such as hospital emergencies and the number of hospitalizations. On the other hand, participants no longer had recourse to specific social services for the homeless population, such as street teams or the dining rooms and considerably reduced the use of the Social Emergency Service. The participants also point out that their vision of the future is more positive and that they now feel they have the conditions to start or resume other activities and projects, in particular employment.

Our group designed a prospective study to assess and compare the financial costs and benefits in Aveiro of a Housing First model in comparison with the Usual model.

2 Description of the problem

Florinhas do Vouga provided some detailed information and have outlined the typical time-line a homeless person goes through in the U and HF models; the different accommodation types available; and several associated
costs.

Let us first consider the usual case. If a homeless individual seeks assistance, they will generally first be given a bed in the emergency homeless shelter. Note that the shelter opens only for the night and only has capacity for 10 men. Once people are in the system, the organisation will assist them in claiming for financial benefits. In order to qualify for such benefits, they must satisfy certain conditions, including regular health checks etc. In the experience of Florinhas do Vouga, it is extremely difficult for a person living on the streets to fulfil all of the criteria required to obtain and maintain these benefits. If they do, the process will typically take around 6 months. Once a homeless person receives these governmental benefits, they can move to a more permanent hostel. Part of their money will be used to pay for the hostel but Florinhas do Vouga will bear the majority of the costs. Ultimately, an individual will become self sufficient or, more likely, will return to the streets or the shelter.

In the Housing First model, the idea is that an individual will stay in the shelter for a short period of time, only until a house becomes available for them, and then move to a permanent home. There, it will be easier for them to satisfy the extensive criteria for receiving government benefits. Once they receive these benefits (as with individuals staying in a hostel in the usual case) it is expected that individuals contribute 30% of their monthly income towards overhead costs. They will continue to remain in the house until they can be self sufficient. In some cases, for example for the elderly or those with severe mental health problems, this may be for the rest of their lives.

A schematic of the timelines of both the Usual and Housing First models are shown in Figure 1 for a time period of 18 months. We consider that at 6 months the individual will start to receive some government benefits allowing the individual to pay for a hostel or, when in a house, to share the house expenses.

In both of these approaches, at any one time, a homeless person will either reside on the streets, in a shelter, in a hostel or in a house. The costs incurred by the organisation, include accommodation, utilities (such as domestic consumption of water and electricity when in a house), food, hygiene (when not in a house individuals need to use the laundry and bathing facilities for personal hygiene) and support services. Some respective associated costs are displayed in the Table 1. When individuals are not in a house the support services are provided by a direct intervention team which gives access to specific social services for the homeless population. In this context the support is institutionalized, not flexible and not individualized. When an individual is in a house the available support services are flexible, individualized, voluntary and oriented according to the needs and objectives of the individuals. These services are provided in the context of the home (some home visits per month) and in other community contexts, in order
**Figure 1:** Typical timeline of accommodation used by a homeless person in the Usual and Housing First cases.

<table>
<thead>
<tr>
<th>Item</th>
<th>Street</th>
<th>Shelter</th>
<th>Hostel</th>
<th>House</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accommodation</td>
<td>400</td>
<td>558</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Utilities</td>
<td></td>
<td></td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Hygiene</td>
<td>35.76</td>
<td>35.76</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Support services</td>
<td>58.35</td>
<td>58.35</td>
<td>58.35</td>
<td>20</td>
</tr>
<tr>
<td>Total amount</td>
<td>214.11</td>
<td>614.11</td>
<td>756.35</td>
<td>480</td>
</tr>
</tbody>
</table>

Table 1: Costs per person, per month (€).

Unsurprisingly, in direct costs to the organisation, it is cheapest to provide support in the management and maintenance of homes, in connection with community resources and in the implementation of individual projects. Nevertheless, it is important to note that, the next cheapest option per person, per month is to accommodate an individual in a house.
3 Solution approaches to the problem

3.1 Considering different scenarios

After the first talk with the people from Florinhas do Vouga we set up some scenarios to allow for an easy calculation of some costs. We consider four scenarios. The number of individuals in each scenario and each form of accommodation is also described in Table 2.

Table 2: Number of individuals in each form of accommodation in each scenario.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Street</th>
<th>Shelter</th>
<th>Hostel</th>
<th>House</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1 (Usual)</td>
<td>42</td>
<td>10</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 2 (Build Shelter)</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Scenario 3 (Usual + Housing First)</td>
<td>30</td>
<td>10</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Scenario 4 (Housing First)</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>30</td>
</tr>
</tbody>
</table>

As a first approach we made some calculations with those scenarios and using the costs displayed in Table 1. In Table 3 we present the results for each scenario per month, per year and for three years.

Table 3: Costs for each scenario for different time periods (€).

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Per Month</th>
<th>Per Year</th>
<th>For 3 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>21184.52</td>
<td>254214.20</td>
<td>762642.70</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>26269.00</td>
<td>315228.00</td>
<td>945684.00</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>22164.40</td>
<td>265972.80</td>
<td>797918.40</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>20823.30</td>
<td>249879.60</td>
<td>749638.80</td>
</tr>
</tbody>
</table>

Our conclusion is that Scenario 4, where most of the people are in the Housing First model, is the one with the lowest costs.

3.2 Considering scenarios which change over time

Considering that this is a dynamic process and that the homeless people move over time between accommodation types with different costs we concluded that a useful mid-term metric would be the number of people in each accommodation type, and the corresponding costs, over a ten month period. We set up a different scenario for each month. We consider that at month 1 we have the Usual model. Thus we consider that people are in the street, in the shelter, in the hostel and nobody is in a house. The Housing First model is being implemented, therefore the number of people in a house is increasing during the considered period of 10 months. This data is given in Table 4.
For each month (in a line) we display the number of people considered in each accommodation type (in a column) and at the last column the total cost. The data given in Table 4 give supporting evidence that the Housing First model is less expensive than the Usual model in the long term.

Table 4: Scenario changes over time.

<table>
<thead>
<tr>
<th>Month</th>
<th>Street</th>
<th>Shelter</th>
<th>Hostel</th>
<th>House</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>10</td>
<td>20</td>
<td>0</td>
<td>27691.40</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>30350.30</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>10</td>
<td>20</td>
<td>15</td>
<td>31679.75</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>20</td>
<td>33009.20</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>30</td>
<td>32956.90</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>35</td>
<td>31575.15</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>40</td>
<td>30193.40</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>10</td>
<td>0</td>
<td>45</td>
<td>28811.65</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>50</td>
<td>28141.10</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>55</td>
<td>27470.55</td>
</tr>
</tbody>
</table>

Next we will consider a Markov chain model, which we use to model the movement between accommodation.

3.3 Using a Markov chain model

The cost each month of helping homeless people in Aveiro is intrinsically a dynamic process. People will move between their accommodation type. For example an individual may move from living on the street to living in a shelter, then after a few months move back to the street, move back to the shelter again, and after a while move to a hostel. Such a stochastic process of people moving between states, with different associated costs, can be described mathematically using a Markov chain [5].

3.3.1 Definition of a Markov chain

Mathematically, for a positive integer $t$, a sequence of random variables $Y_1, Y_2, ..., Y_t, Y_{t+1}$ is a Markov chain if it has the property

$$P(Y_{t+1} = y_{t+1}|Y_t = y_t, ..., Y_0 = y_0) = P(Y_{t+1} = y_{t+1}|Y_t = y_t).$$

(1)

Thus given that a random variable takes a particular value, the probability of what the “next” random variable take is only dependent on the current value, and not on the previous history, i.e. “the path by which we reached the current state”. A Markov Chain is a stochastic process describing the behavior of a system changing between states over some period of time. We use this mathematical framework to describe the probability of people moving between different types of accommodation each month.
3.3.2 Markov chain for accommodation types

Let \( s_1, s_2, \ldots, s_5 \) be the possible accommodation states:

- \( s_1 \) - on the street;
- \( s_2 \) - in the shelter;
- \( s_3 \) - in a hostel;
- \( s_4 \) - in a house provided by a housing first scheme “House”
- \( s_5 \) - in independent, self-supported accommodation “Success”.

The presence of the state \( s_5 \), Success, is to allow possibility for people to become self-sufficient, not needing any welfare or housing financial support. Clearly long term costs will reduce if people can be transferred from dependent living to self-sufficient living.

Let \( Y_t \) be the random variable taking value \( y_t \) in \( \{ s_1, s_2, s_3, s_4, s_5 \} \) and representing the state of an individual at time \( t \). We track the vector \( x(t) = (x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)) \), which gives the number of people in each state according to

\[
x_i(t) = 60\mathbb{P}(Y_t = s_i), \quad i = 1, \ldots, 5.
\]  

Note that the benefit of this model is that it tracks the expected amount of people in each state at each time, that is gives a good estimate of the average likely behaviour. It does not, however, give an individual realisation of what could happen. An alternative approach which does track potential realisations is discussed in Section 3.4.

We have a Markov transition matrix to determine the probability of each individual moving between each of the states \( (s_1 \equiv \text{Street}, s_2 \equiv \text{Shelter}, s_3 \equiv \text{Hostel}, s_4 \equiv \text{House}, s_5 \equiv \text{Success}) \). This assumes that these probabilities are independent of time. We parameterise this transition matrix \( P \) as

\[
P = \begin{bmatrix}
1 - \alpha & \alpha & 0 & 0 & 0 \\
1 - (\beta + \gamma + \delta) & \beta & \gamma & \delta & 0 \\
1 - (\epsilon + \lambda) & 0 & \epsilon & 0 & \lambda \\
1 - (\theta + \mu) & 0 & 0 & \theta & \mu \\
0 & 0 & 0 & 0 & 1
\end{bmatrix},
\]

with \( \alpha, \beta, \gamma, \delta, \epsilon, \lambda, \theta \in [0, 1] \). For example, the transition probability of a person moving to a shelter next month, given that they are currently living on the street, is given by \( \alpha \), and the transition probability of a person moving to a hostel next month, given that they are currently living on the shelter, is given by \( \gamma \).

As shown in the graph in Figure 2, this does not have connections between all nodes, for instance people do not go directly from the street to a hostel,
but instead go via a shelter. We have chosen non-zero transitions based on discussions with members of the charity, but such an approach can be generalised if other non-zero transitions are possible.

The benefit of this framework is that we can change parameters to represent different cases or scenarios, and then see the influence on the expected monthly cost. Note, the values that are found are expected values only.

The rates $\alpha$ and $\beta$ will be the same for the Usual and Housing First cases, since these capture people moving from the street to the shelter, and vice versa. All other parameters are likely to be different between cases, for example we would expect $\mu > \lambda$ under Housing First, since people are more likely to become financially self-sufficient if they first have a house, than if they are first living in a hostel.

For illustrative purposes we choose the following parameter values.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
<th>$\theta$</th>
<th>$\lambda$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Usual</td>
<td>0.25</td>
<td>0.3</td>
<td>0.20</td>
<td>0</td>
<td>0.4</td>
<td>1</td>
<td>0.01</td>
<td>0</td>
</tr>
<tr>
<td>H.F.</td>
<td>0.25</td>
<td>0.3</td>
<td>0</td>
<td>0.6</td>
<td>1</td>
<td>0.85</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

This has simplified the graph so that people can only move into one of the hostel or the house from the shelter, depending on which case is considered.
In order to allow for a fair comparison we use the same initial conditions, which are that all people start on the street, i.e. $x(0) = (60, 0, 0, 0, 0)$.

With these parameter values and initial conditions we find the following results from computational simulations.

![Comparison of number of people in each accommodation types for Usual case (solid lines) and Housing First case (dashed lines), for 36 months, with parameters listed in Table 5.](image)

Figure 3: Comparison of number of people in each accommodation types for Usual case (solid lines) and Housing First case (dashed lines), for 36 months, with parameters listed in Table 5.

As can be seen in Figure 3 the Housing First approach reduces the number of people living on the street and increases the number of people in independent living, when compared to the Usual case. As shown in Figure 4 although the costs in the mid term (around 6 months) are higher for Housing First, the long term costs are less then the Usual approach. Thus the Housing First approach is preferable in the long run, both in terms of financial cost and the number of people becoming independent, for the parameter values used in this example.

This Markov chain model provides a good framework for approaching this problem, to which data for different situations could be applied. However, there are certain constraints which we have not yet included in these models. Here are some of the aspects to include in future Markov chain based models.

- In the Usual case, individuals stay some time in the shelter before they can move to a hostel. When modelling this it is reasonable to take this time to be 6 months.
- The shelter only has capacity for 10 people (men).
Figure 4: Comparison of costs for Usual case (blue) and Housing First case (red), for 36 months, with parameters listed in Table 5.

The effect of the first constraint could be roughly illustrated by merely reducing the transition probability from shelter to hostel in the Usual case. A more sophisticated approach would be to incorporate queueing theory in this Markov chain approach.

3.4 Random walk direct simulations

In modelling homelessness, there are various constraints on accommodation provision, such as those discussed at the end of Section 3.3.2. In this section we consider 3 cases with the following constraints:

- **Usual 1**: Only 10 spaces in the shelter.
- **Usual 2**: Only 10 spaces in the shelter and an individual must spend 6 consecutive months in the shelter to be able to move to the hostel.
- **Housing First**: Only 10 spaces in the shelter.

The simplest approach to model and understand the effect of these is to use direct simulations. We consider each of the $N = 60$ homeless people and take each individual on a random walk over a period of $T = 36$ months.

We consider a stochastic time-dependent process similar to that in Section 3.3. Each individual belongs to a certain state (for example they are in a street). At the next month, each of the individuals can ‘walk’ to either a different state or stay in the same state (in this example, they can either
move to the shelter or stay on the street). For each individual, these actions have associated transition probabilities. The movement between states and allowable transitions are the same as those in the Markov approach, i.e. as in Figure 2.

The difference in this approach lies in the transition probabilities, some illustrative choices of which are shown in Table 6. Here, unlike a Markov approach, the probability an individual moves to a certain state at a given time is dependent on more than just their previous state. We incorporate the constraint that, in the Usual case, individuals must stay in the shelter for 6 consecutive months before they can move to a hostel. To do so, we have to include memory of $C_n$, the number of consecutive months person $n$ has spent in the shelter. For example, in Table 6 we assume that, if an individual is in a shelter but has been there for less than 6 consecutive months, they cannot move to a hostel: they have probability $\gamma = 0$ of moving to a hostel, probability $\beta = 0.8$ of staying in the shelter and probability $1 - \beta - \gamma = 0.2$ of moving to the street. However, if an individual is in a shelter and is at their sixth consecutive month there, they either move to the hostel or go to the streets: there is probability $\beta = 0$ of staying in the shelter, probability $\gamma = 0.9$ of moving to a hostel and probability $1 - \beta - \gamma = 0.1$ of going back to the street.

In addition, because of the limitations of homeless provision, the state of certain individuals affects the others. At any given month, we assume that people come to the shelter successively and the first 10 get beds. We let $B_{n,t}$ be the number of beds occupied at time $t$ months, when person $n$ comes to the shelter. If person $n = n_b$ gets the tenth bed in the shelter, for individuals $n > n_b$, the probability of staying on the street given they are already on the street is 1.

Note that this is a stochastic process. Suppose there are $N_s$ homeless people living on the street at some time and $\alpha$ is the probability someone on the street goes to a shelter. Then in the Markov approach, we would track the expected number of people who move from the street to the shelter, which is $\alpha N_s$. However, in these random walk direct simulations, each of these $N_s$ people have a probability $\alpha$ they will move from the street to the shelter and we track the sum of each of the individual’s results. Therefore, each simulation of the $N$ people for the period of $T$ months will give different results. We perform 1000 such simulations, which has a total computational cost of approximately 10 seconds.

We assume initially that the number of people in each state $x$ is given by $x(0) = (50, 5, 5, 0, 0)$ in the Usual 1 and Usual 2 cases and by $x(0) = (50, 5, 0, 5, 0)$ in the Housing First case. (Here $x(t)$ gives the number of people at time $t$ in each location, respectively: street, shelter, hostel, house and self-supported accommodation.) Our results are illustrated in Figures 5. We plot the mean, median, lower quartile and upper quartile of the 1000 simulations. There is, in the worse case, approximately a 5 person variation.
Table 6: Transition probabilities corresponding to the graph in Figure 2 for person \( n \) at time \( t \). We consider the Usual and Housing First cases with the constraint that the shelter can only occupy 10 people. There are two Usual cases; in the Usual 2 case we specify that an individual must spend 6 consecutive months in a shelter to be able to move to a hostel whilst in the Usual 1 case this constraint is relaxed. Here \( B_{n,t} \) is the number of beds occupied in the shelter at time \( t \) months, when person \( n \) comes to the shelter and \( C_n \) is the number of consecutive months person \( n \) has spent in the shelter.

<table>
<thead>
<tr>
<th></th>
<th>Usual 1</th>
<th>Usual 2</th>
<th>HF</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \begin{cases} 0.15, B_{n,t} &lt; 10 \ 1, \quad B_{n,t} \geq 10 \end{cases} )</td>
<td>( \begin{cases} 0.15, B_{n,t} &lt; 10 \ 1, \quad B_{n,t} \geq 10 \end{cases} )</td>
<td>( \begin{cases} 0.15, B_{n,t} &lt; 10 \ 1, \quad B_{n,t} \geq 10 \end{cases} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.8</td>
<td>( \begin{cases} 0.8, C_n &lt; 6 \ 0, \quad C_n \geq 6 \end{cases} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.1</td>
<td>( \begin{cases} 0, \quad C_n &lt; 6 \ 0.9, \quad C_n \geq 6 \end{cases} )</td>
<td>0</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0</td>
<td>0</td>
<td>0.6</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0</td>
<td>0</td>
<td>0.85</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.1</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

in the quartiles from the mean or median over these 1000 simulations.

Figures 5(a)–(b) correspond to the Usual case with the constraint that the shelter only has 10 beds. We have used almost the same transition probabilities to create these figures but Figure 5(b) includes the constraint that an individual must spend 6 consecutive months in the shelter to move to a hostel. We see that this constraint makes a significant impact on the results of our model - it makes it very difficult for homeless people to progress to the hostel so they can become independent and ‘successful.’ This suggests that, the time and the strict conditions for an individual to receive financial benefits and thus move to a hostel, is a large barrier to a homeless person becoming successful.

We compare the Usual 2 model (with both constraints) in Figure 5(b) with the Housing First case in Figure 5(c). The housing first model has a much greater decrease in the number of people on the streets and increase in the number of self-sufficient, ‘successful’ people. Note that the number of homeless in houses increases initially and later decreases, as those individuals who have benefited from the housing first project have become self sufficient. Furthermore, since everyone in the shelter has the option to al-
most immediately move into a house, the demand on the shelter decreases and, even though there is a 10 bed constraint in the shelter, this is sufficient for the demand.

We couple the mean of our results of the number of people in each state during the 36 month time period with the costs from Table 1. This yields the costs for the Usual model (the Usual 2 model with both constraints) and Housing First models, as shown in Figure 6. We see there is an initial high cost to the Housing First model but in the long term, it is considerably cheaper than the Usual model. Note this qualitative behaviour is similar to more simple Markov chain models, demonstrating the validity of both approaches.

4 Conclusions and recommendations

Upon considering the different mathematical models discussed we can conclude that the Housing First approach is cheaper than the Usual model particularly in the long term.

We modelled the number of people in each accommodation state and used fixed monthly costs for each state to determine total cost. In Section 3.1, we assume the number of people in each state is constant. Our estimates of cost shows that the Housing First model is cheaper. We then use a Markov chain approach to model the changing number of people in each state. This more involved model suggests that the Housing First model is more expensive in the short term but is less costly in the longer term. Including further constraints in a direct simulation of the state of individuals supports the findings of the Markov chain model, demonstrating the validity of both modelling approaches.

We recommend Florinhas do Vouga implement the Housing First approach, as this will not only help to move people off the street and into independent living but will also reduce financial costs in the long term.

5 Suggestions for further work

We make the following suggestions for further research:

- Finding data to obtain better estimates of transition probabilities in both the Markov chain model and the direct simulations.
- Extending the Markov chain model by using queueing theory to incorporate further constraints, such as the limit on the number of people that can stay in the shelter at any one time.
- Using agent based models such as [2], which provide a framework to explore including the 6 month waiting period before people can move
Figure 5: Number of people in each accommodation state from random walk direct simulations. The solid line is the mean of 1000 simulations. The dotted lines are, respectively, the lower quartile, median and upper quartile. (a) Usual 1 case without constraint that only 10 people in shelter (b) Usual 2 case with constraint that people have to spend 6 months consecutively in a shelter to be able to go to a hostel (c) Housing First case with the constraint that only 10 people in shelter.
Figure 6: Comparison of costs each month for Usual 2 and Housing First cases over a 36 month period.

from the shelter to a hostel.

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