Short-Term Scientific Mission

Report

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The Short-Term Scientific Mission funding was used cover the expenses of an academic research visit from February 24th to March 25th of 2016 to the Mathematical Institute at the University of Oxford, United Kingdom. The purpose of the research visit was to advance the ongoing collaboration with Professor Batty and Doctor David Seifert on the long-time behavior of coupled partial differential equations and of infinite systems of coupled differential equations. This collaboration was begun in March 2015 during the time I was visiting University of Oxford for two months as a guest of Professor Batty. The current STSM funding was crucial to the realization of the research visit in March 2016. The research collaboration will continue after the completion of the STSM.

1 Description of the Scientific Activities

We used most of the time of the STSM to extend and generalize our earlier results on the behaviour of infinite systems of linear differential equations. We concentrated on studying the solutions of systems of the form

$$\dot{x}_k(t) = A_0 x_k(t) + A_1 x_{k-1}(t), \qquad x_k(0) = x_{k0} \in \mathbb{C}^m, \qquad k \in \mathbb{Z}, \tag{1}$$

where $A_0, A_1 \in \mathbb{C}^{m\times m}$ are constant matrices. Such systems can be used to approximate the dynamics of long chains of cars on a highway [5] or large groups of interacting robots [3]. In our earlier research [4] we have presented sufficient conditions on the matrices A_0 and A_1 to guarantee stability of the system in the sense that $\sum_{k\in\mathbb{Z}} \|x_k(t)\|^p \to 0$ as $t \to \infty$, where $1 \leq p < \infty$, or alternatively $\sup_{k\in\mathbb{Z}} \|x(t)\| \to 0$ as $t \to \infty$.

During my short-term scientific mission to Oxford we were able to improve our earlier conditions and to extend our results to infinite systems with two-sided coupling

$$\dot{x}_k(t) = A_{-1} x_{k-1}(t) + A_0 x_k(t) + A_1 x_{k+1}(t), \qquad x_k(0) = x_{k0}, \qquad k \in \mathbb{Z}, \tag{2}$$

with constant matrices $A_0, A_1, A_{-1} \in \mathbb{C}^{m\times m}$. Systems of this form can be used to, for example, study a control problem where the objective is to balance the distances between a vehicle from its two neighbouring vehicles in a long queue of cars on a highway. In addition, we extended our results on the stability of systems (1) and (2) to discrete-time systems. The following manuscripts are being prepared based on the results:

Paunonen, L., and Seifert, D. *Decay rates for the infinite robot rendezvous problem*

and

Paunonen, L., and Seifert, D. *Stability of discrete time spatially invariant systems.*

In addition to studying infinite systems of ordinary differential equations, we also began to develop theory for the study of infinite systems of connected partial differential equations. The theory extends our earlier results on coupled wave and heat equations [2] to more complex configurations of partial differential equations. The theory can be used to study the behaviour of energy fluid–structure and heat–structure interactions for systems consisting of a large number of components [6, 1]. The situations that we in particular considered were (i) infinite sequence of linear heat equations with one-sided coupling inside the domain (ii) infinite sequence of heat equations with coupling through the boundary and (iii) infinite sequence of alternating heat and wave equations with boundary coupling. For each of these systems we studied the spectral properties of the full infinite system. In our future work we will continue the research by considering the uniform boundedness and decay rates of the energy in the situations (i)–(iii). In the future our aim is to also generalize our theory concerning chains of PDEs to the study of more general *networks* of partial differential equations. Such systems of equations can be used to study, e.g., flows and distribution of heat and materials in networks of pipes.

References

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